Hybrid games:

Integrating cooperative and noncooperative interactions in strategy analysis

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Abstract: In the formal analysis of value capture, with the exception of biform games, strategy scholars typically employ cooperative game theory. Yet, agents frequently bargain in the shadow of non-negotiated interactions, which may stem from externalities or from transactions governed by protocols. Such interactions may occur if negotiations fail, in which case they determine what an agent can capture acting on its own, or they may occur between groups of agents that negotiate within their respective groups. In either case, understanding the non-negotiated interactions is required to analyze the associated cooperative game. I propose the concepts of negotiation groups and hybrid games to study such combinations of cooperative and non-cooperative interactions. Applications demonstrate how the nature and intensity of the non-negotiated interactions determine value capture. I also show that biform hybrid games feature a novel type of value-based business strategy in which the stage-one actions do not directly affect the agents' positions but impact the non-negotiated interaction between them.

Keywords: Value capture theory, cooperative game theory, noncooperative game theory, bargaining, coalitions, core, characteristic function

1. Introduction

The military strategist von Clausewitz (1832) famously stated that war is the continuation of politics by other means. Similarly, in the context of business strategy, if negotiations fail then firms will often interact in other, less cooperative ways. This includes competing on the market instead of merging, fighting a standards war instead of agreeing on a common technology, or engaging in patent litigation instead of closing a licensing deal. When agents negotiate, they do so in the shadow of these potential non-negotiated interactions (NNIs). Agents may also negotiate among each other while facing, as a group, NNIs with outside groups. These interactions affect the value that each subgroup can claim in negotiations. For example, more intense competition between gaming platforms may strengthen, in negotiations within a platform, the position of a game developer vis-à-vis the console maker.

This paper focuses on combinations of negotiated and non-negotiated interactions as sketched above. I propose the integration of elements of noncooperative game theory into strategy analysis based on cooperative game theory, or "value capture theory," as recently reviewed and synthesized by Gans and Ryall (2017).

In formal analyses of the strategic interaction between firms, researchers have employed noncooperative as well as cooperative game theory. For several decades after von Neumann and Morgenstern's (1944) seminal work, the literature most widely employed noncooperative game theory analysis, a state of affairs that has been critiqued by scholars like Brandenburger and Stuart (1996) and, in the field of economics, by Maskin (2016). Since the 1990s, cooperative games have gained popularity in management research, starting with Brandenburger and Stuart's (1996) analysis of value based business strategies. Scholars have used cooperative game theory to study the general determinants of value capture (MacDonald and Ryall 2004, Montez et al. 2018), the advantages of occupying a broker position in a network (Ryall and Sorenson 2007), the effects of bilateral bargaining on surplus division (de Fontenay and Gans 2014), and the measures that agents can take, in the first stage of a "biform game" to prepare for subsequent negotiations (Brandenburger and Stuart, 2007). Other studies have addressed more specific settings and questions such as buyer-supplier relationships (Chatain 2011, Chatain and Zemsky 2011, Obloj and Zemsky 2015), outsourcing (de Fontenay and Gans 2008), new market entry (MacDonald and Ryall 2018), vertical integration (de Fontenay and Gans 2005), and the role of consumer demand (Adner and Zemsky 2006).

Most of the above studies exclusively employ cooperative game theory (with the exceptions of Brandenburger and Stuart (2007) and de Fontenay and Gans (2005, 2014)). Yet, NNIs matter in the real world, and should be modeled appropriately in formal strategy analysis. Two types of NNIs deserve consideration: potential NNIs that occur in the event that negotiations fail, and realized NNIs that occur between agents or groups that will not, or cannot, negotiate with each other. I discuss both types in turn.

The value that an agent or a group of agents could capture alone is an important factor in any negotiation. Sometimes, this value is independent of the actions of outsiders, in which case the agent simply needs to select its optimal action. In general, however, it will depend on the actions of other agents. For instance, if licensing negotiations between a biotech firm and a pharmaceutical corporation fail, the former may enter the product market and compete with the incumbent, or join forces with a competing corporation. To analyze such situations, von Neumann and Morgenstern (1944) assume a maximin rule, which is conservative but not necessarily realistic. More recent studies assume that the focal group plays a noncooperative game with the agents outside of it

(Zhao 1992, Carraro and Siniscalco 1993, Chander and Tulkens 1997, Ray and Vohra 1997), which yields a natural payoff for the group if the game has a unique Nash equilibrium.

Actually occurring NNIs also matter for value capture. In a cooperative interaction, agents may be organized into groups in such a way that they negotiate with others in the same group, but not with outsiders. For instance, a retailer and its suppliers conduct negotiations with each other, but not with consumers; instead, the retailer posts prices and consumers decide whether and how much to buy. In this example, the outside agents (consumers) are passive in the sense of lacking strategic interaction with the negotiation group. Thus, the NNI results in an optimization problem for the retailer rather than a noncooperative game. An example of an NNI with strategic interaction are the externalities that the retailer exerts on firms located nearby, on competitors but also on complementors such as gas stations. Again, noncooperative games provide an appropriate approach. Montez et al. (2018) model groups unconnected by negotiations, which they refer to as value networks. The authors analyze how the option to join a different group gives an agent leverage in negotiating with its current group. They do not, however, address NNIs between the groups.

To integrate cooperative and noncooperative game theory for strategy analysis, I propose the concept of *hybrid games*. The player set N is divided into k *negotiation groups* $N_1, \ldots, N_k$ such that the players within one group can negotiate with each other, while players in different groups cannot.[1] If there is only one negotiation group, $N_1 = N$, then the value $v(J)$ that a subgroup $J \subset N$

[1] This definition parallels that of the "focal value partition" by Montez et al. (2018: 2720). The difference is, however, that the elements of a focal value partition are "transaction networks", i.e., sets of agents connected through transactions. In contrast, a negotiation group is defined by negotiation linkages between its members. I will elaborate on this distinction in Section 3.

can capture is given by its payoff in the Nash equilibrium of a noncooperative game played by J and the other players. Those, I assume, organize into a known partition, and the resulting game has a unique Nash equilibrium. If there is more than one negotiation group, then a subgroup $J \subset N_j$ considering acting alone has the additional option of joining one of the other negotiation groups. The value $v(J)$ it can capture is given either, following Zhao's (1992) "hybrid solution" of n-person games, by the Nash equilibrium (assumed to exist and to be unique) of the noncooperative game it plays with the players in $N_j \setminus J$, organized into a known partition, and the other negotiation groups; or, if this is larger, by the value it adds to that negotiation group where it adds most, provided this is less than what it adds to its original negotiation group. Payoffs to negotiation groups are determined by the Nash equilibrium (assumed to exist and to be unique) of the noncooperative game played between them. The calculation of J's value capture in case it switches negotiation groups builds on Montez et al.'s (2018) definition of "direct competitive intensity," generalized to account for strategic interactions between the groups.

With increasing numbers of agents, hybrid games quickly become very complex. However, when applied to settings with few players they are tractable while providing novel insights. The concept captures essential aspects of strategic behavior that arise from the combination of negotiated and non-negotiated interactions. Explicitly modeling the NNIs that determine the value a coalition can capture on its own provides a solid basis for the cooperative game that the agents play. In addition, it allows for the study of environmental changes that affect NNIs, such as increased competition due to deregulation, on negotiation outcomes. Moreover, hybrid games can readily be combined with biform games, thus capturing how actions taken in the first stage of a biform game affect negotiations by modifying the potential NNIs underlying the second stage.

In the following section I discuss in more detail several aspects of value capture theory that call for an integration of noncooperative game theory. In Section 3, I formalize the concept of hybrid games. I illustrate the concept in Section 4 using as examples a Cournot duopoly, competing gaming platforms, and biform games, and conclude in Section 5.

2. Noncooperative games in value capture theory

In this section, I discuss when and how noncooperative games may be productively integrated into value capture theory. I first address under which conditions cooperative vs. noncooperative game theory are best suited to model interactions between economic agents. I then discuss NNIs between negotiation groups and how they can impact negotiations within each group. Finally, I discuss the case of NNIs between a subgroup of a negotiation group acting on its own and the other players.

2.1. Negotiations vs. non-negotiated interactions

One key difference between the two branches of game theory is that cooperative game theory "employs a notion of 'free-form' interaction between players" (Brandenburger and Stuart 1996: 7) while a noncooperative game (based on NNIs) requires assumptions about the "protocol" (Kreps 1990: 92) by which the players choose their actions. Real interactions may fit one or the other model, and both types occur regularly in business life. For example, interactions governed by a protocol occur between a retailer and consumers, between competing firms building up production capacity, and between co-located firms exerting demand externalities on each other. "Free-form" negotiations happen between most contract partners as soon as the deal value is too large and transactions are too infrequent for some pre-defined, protocol-based interaction.

A consequence of cooperative games requiring fewer assumptions is that they make less precise predictions: frequently, the core (the most widely used solution concept) contains a continuous set of solutions rather than a single allocation, while Nash equilibria in noncooperative games are typically few or even unique. However, while this loss of predictive power may appear disadvantageous, precision is of little value if it is spurious. Furthermore, this feature makes it possible to isolate the effects of competition from those of "persuasive resources" (Gans and Ryall 2017) and to introduce a novel concept of competitive intensity (Makowski 1980, Ostroy 1980, Montez et al. 2018).

A second, related, difference between the two branches of game theory is that in cooperative games players can negotiate about the actions they take and the division of the value created, and can pin down their agreement in an enforceable contract. This possibility usually ensures that a group of players attains the maximum value that their sets of available actions allow, and thus avoids inefficient outcomes like those that result from monopolistic pricing.[2] As Brandenburger and Stuart (1996: 18) put it, cooperative game theory defines "the size of the overall pie […] under an assumption of 'maximal' flows of resources from suppliers to firms and of products from firms to buyers […]" This assumption in general does not hold for NNIs, where inefficient outcomes are the norm rather than the exception. Consider, for example, the problem of deadweight-loss if a seller posts a single price for all potential buyers.

As a consequence, results from cooperative game theory may be misleading when applied to groups of agents that are not all connected through negotiated interactions. A case in point is the "added value" (Brandenburger and Stuart 1996) of a player A, defined as the increase in the value

[2] Ray and Vohra (1997) point out exceptions to this rule.

created by all players together when A joins. With unrestricted bargaining, one can conclude that A can capture at most its added value. Otherwise, however, it may be that A's joining the group shifts the value distribution among the other members in such a way that A can capture more than its added value (cf. Brandenburger and Stuart 1996: 19)—for example, if its contribution has a substitutive relationship with that of some other network member. Thus, the calculation of the maximal amount a player can capture requires an understanding of the—potentially noncooperative—interactions among the players.

We see that, just as imposing a specific protocol on interactions that are actually free-form negotiations leads to fallacious results, it is equally inappropriate to assume unrestricted bargaining when in fact a protocol exists. For instance, it makes little sense to model a prisoner's dilemma-type situation as a cooperative game, since it is precisely the impossibility to jointly commit to a set of actions that creates the dilemma and the resulting inefficiency.

The above discussion suggests to allow noncooperative games a more prominent place in value capture theory. Brandenburger and Stuart (2007) have pioneered such combinations with the concept of biform games, where agents in a non-cooperative stage take actions that affect their positions in the subsequent cooperative stage. However, real-world combinations of the two types of interactions may differ from the specific timing structure of biform games. In particular, they may occur (1) if a group of agents that negotiate among each other faces NNIs with other groups, and (2) if negotiating agents threaten to revert to some form of NNI in case no agreement is reached. I discuss both cases in turn.

2.2. Non-negotiated interactions between negotiation groups

Negotiations among the members of a negotiation group are usually analyzed without making explicit their linkages to outside agents. Brandenburger and Stuart (1996: 9) describe the underlying assumption in a situation where a triad of manufacturer, buyer, and supplier constitutes the negotiation group: "[…] in arriving at their willingness-to-pay and opportunity-cost numbers, the buyer and supplier were assumed to have access to well-defined prices elsewhere in the economy. […] Loosely speaking, the bargaining problem outside the game under consideration is imagined to have already been solved."

However, if outside agents react to actions by the focal negotiation group, then this assumption is no longer justified. In such a case, reactions by outsiders will in general affect what the focal group as a whole can capture, what each subgroup can secure, and to what extent the threat to join an outside group gives a subgroup leverage in its negotiations. I discuss the three points in turn.

The value captured by the focal group as a whole will depend on the interaction between the group and the outside agents. For example, if a manufacturer and its suppliers negotiate to achieve a price reduction of a consumer product, a competing manufacturer may in turn decrease its price, thus reducing the focal group's value capture compared to a situation without competitive reaction. Such situations require analyzing the NNI between the focal group and outside groups in order to understand the negotiations within the focal group. In the field of mathematical game theory, Zhao (1992) was the first to study this situation.[3]

Similarly, the value that a subgroup of the focal group can secure may depend on NNIs with outsiders. As an illustration, imagine two competing shopping malls, A and B. In each mall, the

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[3] Zhao (1992) assumes that agents are exogenously partitioned into groups that play a noncooperative game while their agents play cooperative games internally. His concept of "hybrid solutions" of games in normal form is thus intermediate between the cooperative solution (one single group) and the non-cooperative solution (each agent a group). Zhao's (1992) main point is to show conditions for the existence of hybrid solutions. The idea to define the characteristic function for each player set in each partition goes back to Thrall and Lucas (1963), who take the values of the characteristic function as exogenously given.

stores negotiate about joint marketing efforts that benefit the mall as a whole. If a Store S in Mall A acts on its own—i.e., does not become party to a joint agreement on marketing measures—then due to the positive externalities of marketing activities on the other stores in A it will invest less, and so will the other stores. As a result, the mall overall is weakened relative to B, to an extent determined by the intensity of the competition (a form of NNI) between the malls. This competition also affects the value that S can capture when acting on its own.

For a subgroup of a negotiation group that faces NNIs with outside agents, "acting on its own" may comprise the option to join an outside group. In this context, Gans and Ryall (2017: 18-20) discuss the example of two value networks, defining a value network as "a collection of agents connected to one another via chains of transactions […]." They consider a firm (F) that is initially part of Network A. By defecting, F reduces A's value capture by $50. In turn, by joining Network B it increases the value that B captures by $30. Assuming, in keeping with the logic of coalitional games, intense competition between the networks for F, F can secure at least $30. Montez et al. (2018) refer to this measure as F's "direct competitive intensity."

Again, NNIs between the networks may come in. To stick with the above example, F's leaving Network A may not only reduce A's value capture, but may also increase B's—even without F joining B—if F's defection weakens A's position in competition with B. To what extent this is the case depends on the type and intensity of the NNI between the networks. Thus, an agent's direct competitive intensity may also depend on NNIs.

The above discussion raises the issue of endogenous network (or coalition) formation, a question addressed by Aumann and Drèze (1974), Aumann and Myerson (1988), Maskin (2016), and a number of other scholars. For the time being, I avoid this type of endogeneity and, apart from considering defections, restrict the analysis to an exogenously given coalitional structure.

2.3. Capturing value "alone"

Compared with the above case of several negotiation groups, the case of a single negotiation group appears deceptively simple. However, even with unrestricted bargaining among all players, bargaining restrictions are inherent to the definition of a cooperative game. To see why, consider the central assumption in cooperative game theory that a group of players can capture a certain value "on its own," which is typically interpreted as meaning that the group members "limit their transactions to one another only" (Gans and Ryall 2017: 24). This seemingly innocuous assumption is problematic in the case of externalities. Even if a group can shun transactions with all other players, the value that it captures may still depend on the outside players' actions. Further, the fallback option in case negotiations fail may not be to forgo transactions entirely, but rather to conduct transactions according to some standard protocol. A case in point are the UK's Brexit negotiations with the European Union, the failure of which would mean that the UK and the Union trade under rules set by the World Trade Organization. In such cases, a sensible mapping of the real-world situation at hand to a game-theoretic model requires an understanding of the interactions of each subgroup of players with the outside players, a question that has been studied in the mathematical literature on game theory by Scarf (1971) and subsequently other scholars. These interactions are genuinely noncooperative, an observation that calls for the application of noncooperative game theory.

Carraro and Siniscalco (1993) as well as Chander and Tulkens (1997) pursue this path in studies of the problem of international pollution (where players exert negative externalities on each other), determining the value that a group of players can capture on its own as its Nash equilibrium payoff in a noncooperative game between the group and the outside players. Carraro and Siniscalco (1993) assume that the outside players act as one coalition, while Chander and

Tulkens (1997) make the opposite assumption, i.e., that they act as individual players. Ray and Vohra (1997) relax this assumption, assuming that the outside players endogenously organize into coalitions. In that case, in order to determine the value that a group of players can capture on its own one needs to analyze, for each possible coalition structure, the noncooperative games between the focal group and all other coalitions, and determine which coalition structure will emerge in equilibrium. While this procedure quickly becomes very complex with increasing numbers of agents, it is tractable in simple, relevant cases. With three firms and externalities, for example, the value that one firm alone can capture depends on the noncooperative game it plays either with the other two firms individually or with the coalition they can form. The coalitional structure that is advantageous to the two firms is assumed to emerge in the equilibrium of the potential game the firms would play if the focal firm was to act on its own.

In this sense, noncooperative games lie at the very heart of cooperative games as soon as there are externalities. If, in a negotiation, a group of players threaten to leave the table and to act on its own, they threaten in effect to play a noncooperative game against the remaining players.

3. Defining hybrid games

I propose a formalization of the concepts and arguments presented above in the definition of *hybrid games*. The purpose of this concept is not to provide a basis for mathematical proofs of the existence and uniqueness of solutions. Rather, it is intended as a tool for analyzing business strategy situations that involve both cooperative and non-negotiated interactions.[4]

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[4] The idea to derive a cooperative game from a game in normal form goes back to von Neumann and Morgenstern (1944). As Scarf (1971: 170) puts it: "These examples illustrate the general proposition that the possibilities open to a coalition should perhaps be viewed as derived from a prior specification of the game in its normal form; that is, in terms of the strategic choices

A hybrid game $H = \left(N, P, \{G_Q\}_{Q\epsilon\mathcal{B}}\right)$ is defined by the player set N, a partition P that sections the player set into disjoint negotiation groups N_1, …, N_k, and the set $\{G_Q\}_{Q\epsilon\mathcal{B}}$ of noncooperative games. Following Montez et al. (2018), I refer to P as the "focal partition." Figure 1 provides an illustrative example with $n = 9$. In the example (Figure 1a), the partition contains three negotiation groups, $P = \{\{1,2,3\}, \{4,5,6,7\}, \{8,9\}\}$.

--- *Figure 1 about here* ---

The games G_Q are indexed by a partition Q of the player set. The relevant partitions are elements of the set $\mathcal{B}$, which comprises two types of partitions, $\mathcal{B} = \mathcal{B}_1 \cup \mathcal{B}_2$. Partitions in $\mathcal{B}_1$ are refinements of P that further divide at most one of the negotiation groups N_j. These partitions model the potential value capture of each subgroup of N_j if it was acting on its own, while taking account of the NNIs that this subgroup is facing with the other subgroups of N_j and with the other negotiation groups N_m, $m \neq j$. The second type of partitions, elements of $\mathcal{B}_2$, are similar to the original partition P, except that a proper subgroup of one of the negotiation groups, $J \subset N_j$, has defected to some other negotiation group, N_m. The respective partition thus consists of $N_j \backslash J$, $N_m \cup J$, and all other negotiation groups N_i, $i \neq j$, $i \neq m$. These partitions capture a subgroup's option—if that option exists—to threaten defection to some other negotiation group, and use this threat in its negotiations with the other group members.

open to the individual players, and their evaluations of the outcomes. von Neumann and Morgenstern attempted to do precisely this […]"

With the above definitions I derive the characteristic function for $J \subseteq N_j$ as follows. First, I assume that if J is acting on its own without joining some other negotiation group, then the players in $N_j \setminus J$ organize into a known partition, $Q_J \in \mathcal{B}_1$.[5] For illustration, Figures 1b and 1c show the coalitions that can form if the one-player coalition $J = \{1\}$ leaves the negotiation group, $N_1 = \{1,2,3\}$. The remaining players in this group may form a single coalition $\{2,3\}$ (Figure 1b) or act as individual players (Figure 1c). Which of these partitions arises depends on the setting; in a Cournot quantity cartel, for example, defection of one player would make it optimal for the others to split up as well. Further, I assume that the noncooperative game G_{Q_J} has a unique Nash equilibrium in pure strategies. I define the value $v_1(J)$ that J can capture by acting on its own, but without joining some other negotiation group, as its payoff in this equilibrium.

Regarding J's option to defect, let $Q_{J,m}$ denote the partition that occurs if J joins N_m. I assume that the game $G_{Q_{J,m}}$ as well as the game G_P played between the original negotiation groups have a unique Nash equilibria in pure strategies. J's added value by joining N_m is then given by the payoff that $N_m \cup J$ receives in the Nash equilibrium of $G_{Q_{J,m}}$ minus what N_m receives in the equilibrium of G_P. I denote the maximum of these added values over all negotiation groups $m \neq j$ as $v_2(J)$, and assume that it is less than J's added value[6] to N_J—a plausible assumption since otherwise the focal partition P would be unstable.

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[5] To keep the formalism simple, I refrain from specifying how Q_J is determined, thus avoiding the thorny issue of endogenous coalition formation (e.g., Maskin 2016). In sufficiently simple and tractable cases, the selection of Q_J will be obvious.

[6] More appropriately, one should speak of J's "subtracted value", i.e., the loss in value for the negotiation group N_j caused by J's defecting.

The characteristic function of the coalition J is then given by the maximum of what it can achieve by acting on its own and by joining some other negotiation group (if that option exists), $v(J) \coloneqq \max\{v_1(J), v_2(J)\}$. Based on $v(.)$, the cooperative game within each negotiation group can be solved.

Hybrid games can become rather complex. In the example shown in Figure 1, one needs to analyze a minimum of 13 noncooperative games if defections are excluded, and an additional 44 if they are allowed.[7] For many practically relevant cases, however, the complexity is manageable. In the simplest case of two firms negotiating in the shadow of an NNI, one needs to define and solve just a single noncooperative game. Or, consider the case of six players, organized into two negotiation groups of three players each, $N_1 = \{1,2,3\}$ and $N_2 = \{4,5,6\}$, and assume that defections are excluded. Then the number of different Nash equilibria to be determined equals nine: one for the focal partition, one for each of the four finer partitions of N_1 ($\{\{1\},\{2\},\{3\}\}$,$\{\{1\},\{2,3\}\}$, $\{\{2\},\{1,3\}\}$, $\{\{3\},\{1,2\}\}$), and one for each of the four finer partitions of N_2. If it is known that the remaining players in N_j form a negotiation group if a single player in N_j acts on its own, then this number is reduced to seven. It is further reduced to four if there is symmetry between N_1 and N_2. This is a manageable degree of complexity, even with six players.

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7 If defections are excluded, the minimum obtains if for each coalition J leaving its negotiation group N_j the remaining players form a single coalition. It is calculated by adding up, over all negotiation groups, the number of possibilities to split the group into two subsets, and adding one for the game played by the focal partition. This calculation yields $3 + 7 + 1 + 1 = 13$. If defections are allowed, then there are two additional games to be considered for each proper subset of each negotiation group, hence 2 × (6 + 14 + 2) = 44.

4. Applications

In the following, I discuss three cases that illustrate the concept of hybrid games. In the first, a Cournot quantity cartel, the focus is on negotiations in the shadow of competition between the negotiators. In the second case, a console manufacturer and a game developer negotiate to coordinate their activities, faced with competition in a marketplace that includes a second gaming platform. The third case combines the concepts of hybrid games and of biform games.

4.1. Cournot quantity cartel

As a simple illustrative example, consider the case of three firms negotiating to form a quantity cartel. The standard Cournot demand function defines the protocol that governs the interaction with buyers, where market price p is given by $p = 1 - q_1 - q_2 - q_3$. The single negotiation group is the set of all three firms, $P = \big\{\{1,2,3\}\big\}$. Since there is only one negotiation group, $\mathcal{B} = \mathcal{B}_1 = \Big\{\big\{\{1\},\{2\},\{3\}\big\},\big\{\{1\},\{2,3\}\big\},\big\{\{2\},\{1,3\}\big\},\big\{\{3\},\{1,2\}\big\}\Big\}$. The hybrid game is thus given by $H = \left(N, P, \{G_Q\}_{Q\epsilon\mathcal{B}_1}\right) = \left(\{1,2,3\},\{\{1,2,3\}\},\{G_Q\}_{Q\epsilon\mathcal{B}_1}\right)$, where the games G_Q are simple Cournot models with the number of players equal to the cardinality of the partition Q.

To calculate the value that a single firm i can capture we need to determine whether, if firm i acts on its own, the remaining firms form a coalition or act individually. In the first case, a standard Cournot duopoly, they would jointly capture $1/9$, and in the second each would capture $1/16$. Thus, they are better off acting individually (as is well known), and so it is plausible to assume that the game firm i plays when acting on its own is a Cournot game with three players, each of which earns $1/16$. The characteristic function of the resulting cooperative game is thus given by $v(\{i\}) = 1/16$, $v(\{i,j\}) = 1/9$ ($i \neq j$), and $v(\{1,2,3\}) = 1/4$. Its core is the set of all allocations (x_1, x_2, x_3) that fulfill $x_j \geq 1/16$ for all j, and $x_1 + x_2 + x_3 = 1/4$.

This simple example illustrates the necessity of understanding the potential noncooperative game that agents would play if they failed to reach an agreement. I now turn to less stylized and more complex cases.

4.2. Competing gaming platforms

Negotiations are particularly fruitful and, unlike cartels, usually permitted if the negotiation partners complement each other. Such situations are common in the ICT industry. As an application from this field, consider the interaction between two game console makers (CM) and two game developers (GD), illustrated in Figure 2. CM 1 and GD 1, and similarly CM 2 and GD 2, develop products that are complementary to each other. I refer to a combination of a console and the complementary games as a bundle. There are a large number of consumers who buy at most one bundle. The console makers ship their products with some games included, such that consumers also value the consoles alone.

The extent to which the agents exploit their potential for value creation depends on the actions they take with respect to product qualities and quantities. Those, in turn, depend on the negotiation and protocol structure that governs their interactions. I discuss four different, increasingly realistic scenarios, depicted in Figures 2a to 2d. Each figure shows, on the left, the actual negotiation and protocol structure. Agents within a box with a full line boundary engage in unrestricted bargaining: they form a negotiation group. A box with a dotted line boundary indicates that a protocol governs the interaction between the players and negotiation groups within it. The right of each figure shows the potential negotiation and protocol structure that would arise if GD 1 left its negotiation group and acted on its own.

Numbers next to the boxes denote the value that the respective agent or negotiation group captures. Comparison with the left part of the figure yields GD 1's added value. Calculating these numbers requires setting up and solving the noncooperative game played in the respective situation. These analyses are omitted for brevity, as the purpose is to focus on the ultimate impact of the NNI.

--- *Figure 2 about here* ---

4.2.1. Negotiation groups of firms and consumers

In Figure 2a, I assume that CM 1, GD 1, and a consumer group (Consumers 1) form a negotiation group.[8] Because of unrestricted bargaining, they will achieve the maximum possible value creation. The same holds for CM 2, GD 2, and the second consumer group. The negotiation groups are symmetric, each creating a value of 30.

If GD 1 left its negotiation group to act on its own, how would it interact with CM 1 and Consumers 1? It may be that GD 1 actually ceases transacting with the other parties, in particular if CM 1 blocks the technical interface between its platform and GD 1's games. However, doing so may not be in CM 1's interest. Thus, it may also be that "acting on its own" means that GD 1 will sell to the consumer group via a protocol, i.e., an NNI.

For illustrative purposes, I assume here that the former case happens and GD 1 ceases all transactions and captures a value of zero. CM 1 and Consumers 1 then capture a reduced value of 15, less than with GD 1's participation, due to the value of GD 1's games and complementarity between console and game. The difference between the two values, 15, is GD 1's added value, which constitutes an upper limit for what GD 1 can capture. In turn, the minimum GD 1 will

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8 I call the potential buyers of one of the product bundles "consumers," and the actual buyers "customers." Consumers who do not buy any product capture a value of zero. Thus, the value captured by one of the consumer groups is the same as the value captured by the corresponding customer group.

contend with is zero, since it cannot capture any value on its own. Since the negotiation and protocol structure does not connect the second to the first negotiation group, the former is unaffected by GD 1's (potential) move.

4.2.2. Negotiation groups of firms and consumers linked through protocol

It is hardly plausible that a large group of consumers takes part in unrestricted bargaining: their interaction with firms is more appropriately modeled as a noncooperative game in which firms post unique prices. Figure 2b shows a negotiation and protocol structure in which CM 1 and GD 1, and CM 2 and GD 2, form negotiation groups, selling to separate groups of consumers. Within each group, the respective partners coordinate their quality and pricing decisions in such a way as to maximize the value they create and capture jointly, internalizing the externalities inherent in complementary products. Quantities are indirectly determined by how consumers react to the firms' pricing. I assume the resulting deadweight loss reduces the overall value created by GD 1, CM 1 and their consumers to 25 (reduced from 30 in Scenario a), with the firms jointly capturing 20 and their consumers 5.

In this structure, if GD 1 acted on its own, then it would still capture value by selling its product to Consumers 1. Since the interaction between GD 1 and the consumers is realistically modeled as governed by a protocol, there is no reason to assume it would terminate if GD 1 acted on its own.[9] In this situation, the overall value created by CM 1, GD 1, and their consumers falls compared to 1b (left) since the firms do not internalize the externalities they exert on each other

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[9] As mentioned earlier, CM 1 could in principle block the interface of its platform to GD 1's games. However, since GD 1 is the only game developer offering games for CM 1's platform and there is no repeated interaction, doing so would not be in CM 1's interest and hence a non-credible threat.

through their product quality and pricing choices. I assume that CM 1 and GD 1 each capture 8 and their consumers, 3, adding up to 19.

What is GD 1's added value in this situation? Looking at the transaction network consisting of CM 1, GD 1, and their consumers, GD 1's added value is equal to 14 (25 minus 11). However, GD 1 is only negotiating with CM 1, not with the consumer group. Thus, the consumers' value increase when GD 1 joins, from 3 to 5, is not subject to the negotiation, and if GD 1 asked for 14 in its negotiation with CM 1, the latter would prefer the value of 8 that it captures alone to the remainder of 20 minus 14. Thus, the relevant upper bound for what GD 1 can capture is its added value to the negotiation group consisting of itself and CM 1. We obtain an added value of 12 (20 minus 8), down from 15 in Scenario a. Furthermore, the minimum that GD 1 will contend with is 8, up from 0 in 1a. The more realistic assumption of a protocol-based interaction with consumers thus narrows down the interval of possible negotiation outcomes for GD 1, from [0,15] in Scenario a to [8,12] in b.

4.2.3. Negotiation groups competing for consumers

In an even more realistic scenario, depicted in Figure 2c, I assume that each consumer interacts with all firms through a protocol, and decides which product bundle to buy, if any. Thus, the two negotiation groups exert externalities (NNIs) on each other via market competition. Each negotiation group, CM 1 plus GD 1 and CM 2 plus GD 2, determines its product qualities and prices in unrestricted internal bargaining, anticipating the consumers' reaction and taking competition from the other coalition into account. The product bundles are horizontally differentiated, such that the sellers retain some market power.

I assume that the resulting value distribution is symmetric, with each negotiation group capturing 18 and its customers capturing 9. Compared to Scenario b, left, each two-firm coalition captures less, 18 instead of 20, due to competition. Each consumer group captures more, 9 instead of 5: lower prices increase consumer surplus, and in addition reduce deadweight loss since more consumers decide to buy.

If GD 1 left the negotiation group with CM 1 and acted on its own (Figure 2c, right), then the same mechanisms are at work as in Scenario b. In particular, the lack of coordination between CM 1 and GD 1 with respect to qualities and pricing will make their product combination less attractive. In this case, however, this reduction in attractiveness will make some consumers switch to the competing product bundle offered by CM 2 and GD 2. Those firms will furthermore adapt their quality and pricing decisions to the changed competitive situation.

As a result, the total value captured by CM 1 and GD 1 is reduced when GD 1 leaves the negotiation group with CM 1, from 18 to 12 in Scenario c, compared to a reduction from 20 to 16 in b. The consumers buying from CM 1 and GD 1 in the right-hand scenario in Figure 2c capture 4, down from 9 in the left-hand scenario, while the consumers buying from the competing coalition capture 12, up from 9. This increase is due to an increase in the number of customers. It does not make up for the decrease in value captured by the other group, from 9 to 4, since overall the lack of coordination between CM 1 and GD 1 harms consumers. The coalition of CM 2 and GD 2 captures 22, up from 18 in the symmetric situation (Figure 2c, left). As for the corresponding consumer groups, this increase does not fully make up for the loss, from 18 to 12, that the other coalition incurs.

How is GD 1's added value determined in this case? As already argued in Scenario b, GD 1 negotiates with CM 1 only, not with their consumers, such that its added value should refer to what the two firms can capture. One obtains an added value of 18 minus 6, equal to 12. As a result, possible negotiation outcomes for GD 1 lie in the interval [6,12].

This scenario underlines why added value should be calculated relative to a negotiation group rather than the industry as a whole. Some of the value of 18 – 6 = 12 that GD 1 adds to CM 1 is due to reduced value capture by the negotiation group consisting of CM 2 and GD 2, which goes down from 22 to 18 when GD 1 joins forces with CM 1. Similarly, the joint value capture of CM 1 and Consumers 1 increases by 17 from 6 + 4 = 10 to 18 + 9 = 27 when GD 1 and CM 1 form a negotiation group, but by the same token the joint value capture of CM 2, GD 2, and Consumers 2 goes down by 7 from 22 + 12 = 34 to 18 + 9 = 27. Thus, GD 1 can stipulate more in its negotiations with CM 1 than the additional value that its joining GD 1 creates on the industry level.

4.2.4. The option of defecting

In the scenario illustrated in Figure 2d, GD 1 has the option of joining the competing platform. Assuming that the games offered by GD 1 and GD 2, respectively, are not perfect substitutes to each other, and ignoring potential costs of establishing compatibility with Platform 2, the new negotiation group consisting of CM 2 and both game developers (Figure 2d, right) will capture more value (in the illustration, 32) than CM 2 and GD 2 in the reference scenario (18). GD 1's added value to Platform 2 thus equals 32 – 18 = 14, its direct competitive intensity (Montez et al., 2018). At the same time, CM 1 alone captures much less than in a negotiation group with GD 1, 1 compared to 18. GD 1's added (or subtracted) value to CM 1 is thus 18 – 1 = 17, and its value capture in negotiations with CM 1 should lie between 14 and 17.

The discussion of value creation and capture in the case of competing gaming platforms demonstrates that understanding the NNIs between the agents is crucial in analyzing cooperative interactions between them. Agents may purposefully manipulate the nature of these NNIs before the hybrid game takes place. Such manipulations can be analyzed using biform games.

4.3. Biform hybrid games

Biform games (Brandenburger and Stuart 2007) and hybrid games both integrate elements of cooperative and noncooperative game theory, but they do so in fundamentally different ways. In this section, I show that combining these approaches is possible and allows for, among other things, a categorization of biform games.

In the first noncooperative stage of a biform game, agents make strategic moves. The actions they take in this stage shape the competitive environment in which, in the second stage, the cooperative interaction takes place. Brandenburger and Stuart (2007) present several examples. In a branded-ingredient game, a supplier invests in raising customers' willingness-to-pay for products containing its branded component, thus changing the setting for the cooperative interaction with the manufacturer in the second stage. In an innovation game, firms can invest in R&D to develop a new generation of their product, thus shaping the negotiations with buyers in the second stage. In a coordination game, three firms independently decide whether to incur the costs of switching to a new technology standard, where the value that each can capture in the subsequent cooperative stage depends on the choices made by the other firms. Finally, in a repositioning game, a firm may invest to raise the perceived quality of its product, thus improving its position in negotiations with buyers.

In all of these examples, actions taken in the noncooperative stage improve the position of the respective agent in the subsequent cooperative interactions. Alternatively, such actions may weaken the position of competing agents, in line with Brandenburger and Stuart's (1996) "value-based business strategies" that target competitors.

Hybrid games open up further possibilities. If the second stage of a biform game is a hybrid game, then actions taken in stage one may affect the interaction itself rather than directly affecting the agents' positions. For example, if acting on its own means firms compete, then a competitively strong firm may improve its position in the potential NNI, and hence the value it can capture alone, by increasing the intensity of competition. In the following, I analyze such a case in detail.

Consider two sellers of horizontally differentiated goods that negotiate to form a price cartel. Following Hotelling's (1929) classic model, buyers are distributed with unit density on the interval [0,1]. Sellers A and B are located at 0 and 1, respectively. Seller X, $X \in \{A, B\}$, has a unit cost of c_X and charges p_X for its product. A buyer located at $x \in [0,1]$ has a willingness-to-pay of $u_A - tx$ for A's product and of $u_B - t(1-x)$ for B's. The maximum value creation per customer of Firm X is thus given by $u_X - c_X$. The parameter t, which measures transportation cost in spatial interpretations of the model, describes customers' loss in utility from not obtaining their most preferred product variant. Smaller t implies a higher intensity of competition. Each potential customer buys either one unit of A's product, one of B's, or none.

I distinguish two cases depending on the parameters. For sufficiently small values of t, the superior firm serves the entire market, while for larger t each firm has a positive market share. For simplicity, I restrict the analysis to parameter ranges where the entire market is being served.[10]

Let $\alpha \coloneqq (u_A - c_A - u_B + c_B)$ denote A's advantage over B with respect to maximum value creation per customer (net of transportation cost), and $\mu \coloneqq (u_A - c_A + u_B - c_B)/2$ the firms' average value creation per customer. Without loss of generality, I assume that Firm A is superior to B, hence $\alpha \geq 0$. All proofs are in the Appendix. In a cartel, the firms' joint profits are given by the following equation:

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[10] If t is large relative to u_A and u_B, then the firms each serve only a "local" market and leave some potential buyers unserved. Since this case is not informative for the example of a biform hybrid game, I exclude it.

$$v(\{A,B\}) = \begin{cases} \mu - t + \frac{\alpha}{2} & : \quad t \leq \frac{\alpha}{2} \\ \mu - \frac{t}{2} + \frac{\alpha^2}{8t} & : \quad t > \frac{\alpha}{2} \end{cases} \tag{1}$$

In these expressions, the respective first term captures joint profits in the symmetric case without accounting for transportation cost ($\alpha = 0$, $t = 0$). The second term accounts for the fact that a seller needs to set its price below the willingness-to-pay of the customer most closely located to it in order to have a positive market share. The third term captures deviations from the symmetric situation. For interior solutions (second line), it is quadratic in α because deviations imply not only differences in the firms' margins, but also a shift in the position of the marginal customer.

The firms' fallback option in case they do not reach an agreement on cartel formation is to compete on price. Again, two cases need to be distinguished.

$$v(\{A\}) = \begin{cases} \alpha - t & : \quad t \leq \frac{\alpha}{3} \\ \frac{(3t+\alpha)^2}{18t} & : \quad t > \frac{\alpha}{3} \end{cases} \qquad v(\{B\}) = \begin{cases} 0 & : \quad t \leq \frac{\alpha}{3} \\ \frac{(3t-\alpha)^2}{18t} & : \quad t > \frac{\alpha}{3} \end{cases} \tag{2}$$

Figure 3 shows that joint value creation (Equation 1) decreases strictly monotonically in t, which is plausible. In competition (Equation 2), the effect of changes in t varies. For small values of t, where A serves the entire market, B's value capture is zero while A's decreases in t. The intuition for this decrease is that, in order to sell to all buyers including the one farthest away from A, the price that A can charge is limited to $u_A - u_B + p_B - t$, which equals $u_A - u_B + c_B - t$ if B prices at marginal cost. This price implies profits for A of $\alpha - t$. For $t > \alpha/3$, increases in t

amount to a reduction in the intensity of competition, such that both firms' value capture increases in t.

--- Figure 3 about here ---

If Firm A has the means to increase the intensity of competition in the noncooperative first stage of the biform game, it may use this power to change the ensuing cooperative game in its favor. Compare value capture at $t = 1$, where $v(\{A, B\}) = 2.13$, $v(\{A\}) = 0.89$, and $v(\{B\}) = 0.22$, to that at $t = 0$, where $v(\{A, B\}) = 3$, $v(\{A\}) = 1$, and $v(\{B\}) = 0$. By shifting t from 1 to 0, Firm A increases overall value capture $v(\{A, B\})$ by $3 - 2.13 = 0.87$, its own minimum residual $v(\{A\})$ by $1 - 0.89 = 0.11$, and its added value $v(\{A, B\}) - v(\{B\})$ by $3 - 1.91 = 1.09$, while decreasing B's minimum residual from 0.22 to 0. Even though the shift to higher competition intensity increases B's added value from 1.24 to 2, it is in every respect favorable to A.

This example nicely demonstrates how value-based business strategies enacted in the first stage of a biform game work in three different ways. They may directly improve the position of the focal firm, which for Firm A means to increase buyers' willingness-to-pay, u_A, or to decrease its cost, c_A. They may also weaken the competitor's position through a reduction of buyers' willingness-to-pay, u_B, or an increase in cost, c_B. To these established value-based business strategies a new type is added in the case of biform hybrid games. By changing the transportation cost parameter, t, a firm may shape to its advantage the competition in the potential NNI overshadowing the negotiation.

5. Discussion and conclusions

I argue that negotiations frequently take place in the shadow of non-negotiated interactions (NNIs). Negotiating parties may face NNIs with an outside negotiation group, or between each other in case they reach no agreement. NNIs may arise due to externalities or to transactions that follow a protocol rather than being freely negotiated. Hybrid games model these NNIs as noncooperative games whose Nash equilibria establish the characteristic function underlying the negotiated interaction. I illustrate the usefulness of the concept using the examples of a Cournot quantity cartel and of competing gaming platforms, each of which requires collaboration between a console maker and a game developer. I furthermore show how hybrid games, combined with biform games, facilitate a new type of value-based business strategy (Brandenburger and Stuart, 1996), one that targets the noncooperative interaction overshadowing the negotiations.

The analysis has several limitations and suggests a number of generalizations. First, I assume existence and uniqueness of a pure strategy Nash equilibrium for each of the noncooperative games within hybrid games. If either condition is violated, the concept requires generalization to mixed-strategy equilibria or to multiple equilibria as arising, for example, in coordination games. I also exclude frictions (Brandenburger and Stuart 1996, Chatain and Zemsky 2011), assuming that within a negotiation group, all agents engage in unrestricted bargaining. Allowing for frictions would generalize the model, and would in particular allow for the analysis of specific positions such as that of a broker (Ryall and Sorenson 2007). Relatedly, I do not address the relevance of how members of a group actually negotiate, i.e., if they engage in pairwise negotiations (as in de Fontenay and Gans 2014) or in larger subgroups, possibly with all group members around the negotiation table. Finally, I assume, as Zhao (1992) and Montez et al. (2018), that negotiation groups (respectively, transaction networks) are exogenously given. The endogenous emergence of negotiation groups is a highly relevant but elusive issue (e.g. Maskin 2016), to which the concept of hybrid games may fruitfully be applied.

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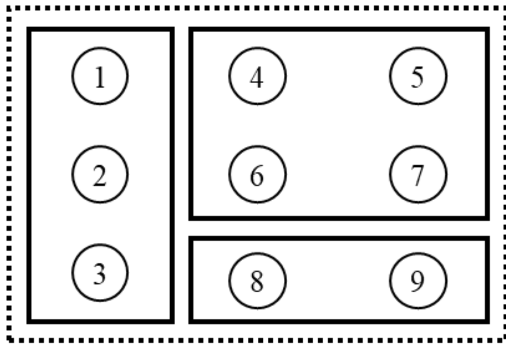
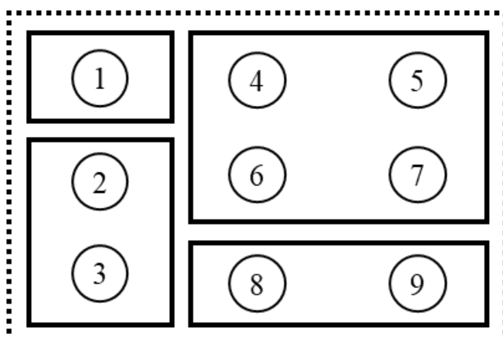
Figures

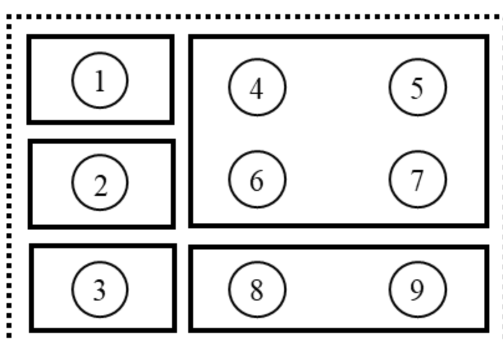
Figure 1: Different negotiation and protocol structures



(a) Realized negotiation situation



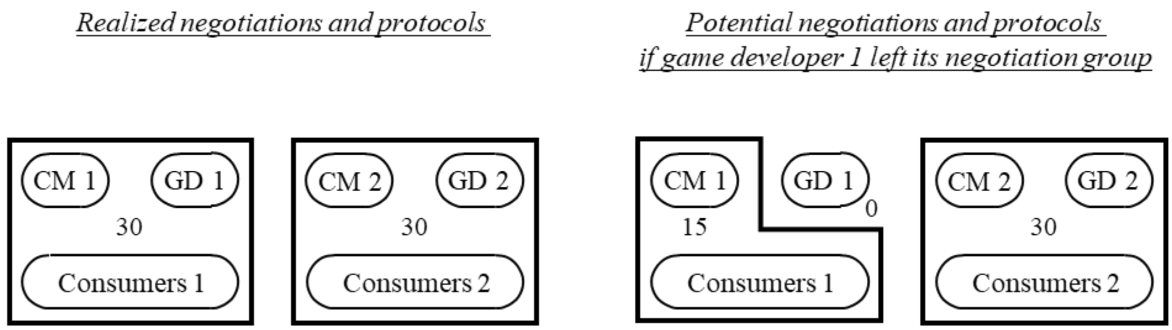
(b) Potential negotiation situation if 2 and 3 form a coalition



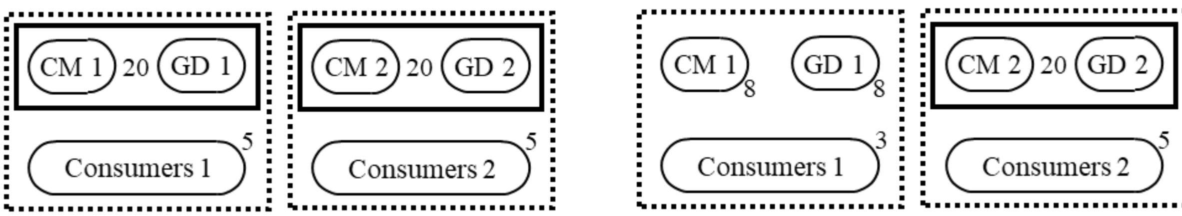
(c) Potential negotiation situation if 2 and 3 act separately

Note: Actors within the same box with a full line boundary (———) form a negotiation group; a dotted line boundary (⋯⋯) indicates that the enclosed negotiation groups interact through a protocol.

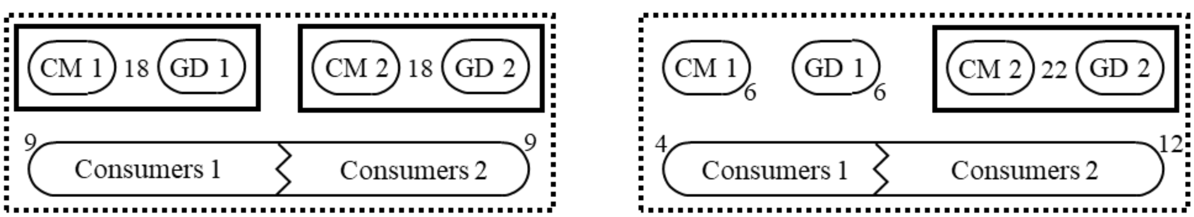
Figure 2: Gaming platforms: Realized and potential negotiation situations



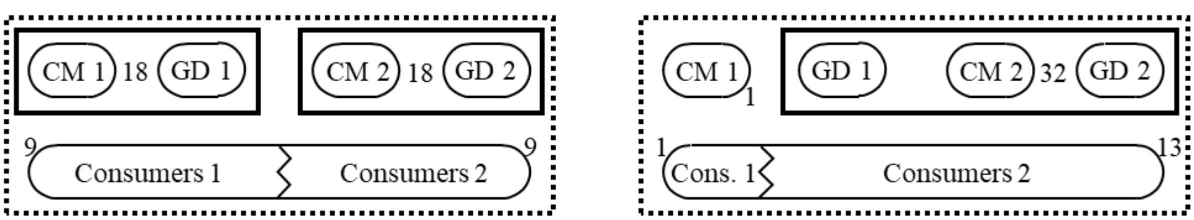
(a) Each console maker forms a negotiation group with one game developer and their consumers.



(b) Each console maker forms a negotiation group with one game developer.
Each consumer interacts with one negotiation group through a protocol.



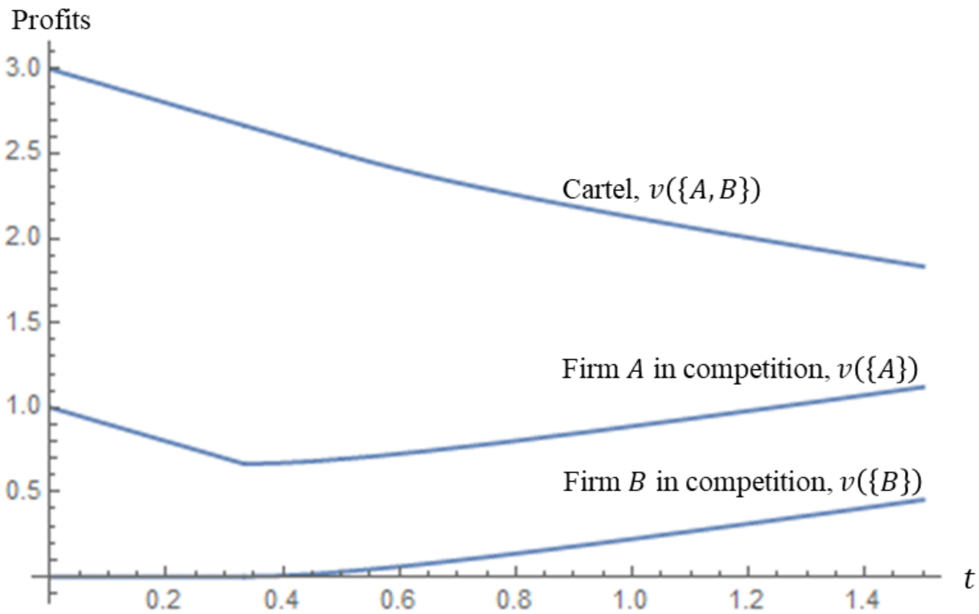
(c) Each console maker forms a negotiation group with one game developer.
Each consumer interacts with both negotiation groups through a protocol.



(d) Each console maker forms a negotiation group with one game developer. GD 1 can defect to Platform 2. Each consumer interacts with both negotiation groups through a protocol.

Note: CM = console maker; GD = game developer. Actors within a box with a full line boundary (———) form a negotiation group; a dotted line boundary (⋯⋯) indicates that the players or negotiation groups within the box interact through a protocol. Numbers denote the value captured by a player or a group.

Figure 3: Profits in a cartel and in competition ($\mu = 2.5$, $\alpha = 1$)



Appendix

Solving the game requires two types of case distinctions. First, between a cartel and price competition; and second, between different ranges of the parameter, t. For very high values of t, it will not be profitable for the firms to serve the entire market. For simplicity, I exclude this case by restricting the analysis to parameter ranges where the entire market is being served. For very small values, the entire market will be served by the superior firm (Firm A); and for intermediate values, both firms make positive sales and together serve the entire market.

Cartel

For medium values of t, where the entire market is served and each firm has a positive market share, the position of the marginal consumer is given by $x^* = (u_A - u_B - p_A + p_B + t)/t$. Maximizing their joint profits, firms will set prices in such a way that the marginal consumer has zero utility from buying either product: $u_A - p_A - tx^* = 0 \wedge u_B - p_B - t(1 - x^*) = 0$. These equations allow to express p_B as a function of p_A, as $p_B = u_A + u_B - p_A - t$. Total profits are given by $x^*(p_A - c_A) + (1 - x^*)(p_B - c_B)$. Substituting the above expressions for x^* and p_B and calculating the derivative with respect to p_A yields the first-order condition, $p_A^* = (3u_A - u_B + c_A - c_B - 2t)/4$. Inserting this and the corresponding term for p_B^* into the profit function and using the parameters α and μ yields $\Pi_C(\alpha, \mu, t) = \mu - t/2 - \alpha^2/(8t)$. This solution holds when the corresponding expression for the marginal consumer, $x^* = \alpha/(4t) + 1/2$, yields a solution within the interval $[0,1]$, which is equivalent to $|\alpha| \leq 2t$ or, since I assume $\alpha > 0$, $\alpha \leq 2t$.

For high values of t, each firm X serves only a local market of the size $(u_X - p_X)/t$, realizing profits of $(p_X - c_X)(u_X - p_X)/t$. Optimizing with respect to p_X yields a market size of

$(2\mu + \alpha)/(4t)$ for A and of $(2\mu - \alpha)/(4t)$ for B. These local markets merge and cover the entire market when their sizes add up to unity, which is the case for $\mu = t$. This condition implies that the interior solution with each firm having a positive market share holds for $\alpha/2 < t < \mu$.

For low values of t, $t < \alpha/2$, the superior firm A serves the entire market, setting p_A such that the consumer at $x = 1$ has zero utility: $u_A - p_A - t = 0$. The profit function equals $p_A - c_A$, such that equilibrium profits are $\Pi_C(\alpha, \mu, t) = \mu - t + \alpha/2$.

Competition

As in the cartel case, for medium values of t with the entire market being served and each firm having a positive market share, the position of the marginal consumer is given by $x^* = (u_A - u_B - p_A + p_B + t)/t$. The firms' profit functions are given by $x^*(p_A - c_A)$ and $(1 - x^*)(p_B - c_B)$, respectively. Differentiating yields the first-order conditions, solving which leads to the equilibrium prices, $p_A^* = t + (u_A - u_B + 2c_A + c_B)/3$ and $p_B^* = t + (u_B - u_A + 2c_B + c_A)/3$, and to the position of the marginal consumer, $(\alpha + 3t)/(6t)$. Inserting these terms into the firms' profit functions yields the equilibrium profits, $\Pi_A^* = (3t + \alpha)^2/(18t)$ and $\Pi_B^* = (3t - \alpha)^2/(18t)$.

These solutions apply as long as the above term for the position of the marginal consumer yields a value within [0,1], which is the case for $t \geq \alpha/3$. If $t < \alpha/3$, then A serves the entire market. To ensure that B does not sell anything, the value of A's product for the buyer at $x = 1$, $u_A - p_A - t$, must be at least as large as that of B's product assuming B prices as marginal cost, $u_B - c_B$. Thus, $p_A^* = u_A - u_B + c_B - t$, and $\Pi_A^* = p_A^* - c_A = \alpha - t$.

The maximal value of t up to which the interior solution calculated above holds is determined as follows. For very large values of t, each firm serves a local market (as in the cartel case), and these markets merge to cover the entire interval $[0,1]$ if $t = \mu$. Thus, for $t > \mu$ the interior solution is not applicable. However, another condition needs to be observed, which is that the marginal consumer must have non-negative utility from buying either product. This condition implies that $t < 2\mu/3$. In Figure 3, with $\mu = 2.5$ and $t \leq 1.5$, it is fulfilled.